Nonperturbative analysis of coupled quantum dots in a phonon bath

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Transport through coupled quantum dots in a phonon bath is studied using the recently developed real-time renormalization-group method. Thereby, the problem can be treated beyond perturbation theory regarding the complete interaction. A reliable solution for the stationary tunnel current is obtained for the case of moderately strong couplings of the dots to the leads and to the phonon bath. Any other parameter is arbitrary, and the complete electron-phonon interaction is taken into account. Experimental results are quantitatively reproduced by taking into account a finite extension of the wavefunctions within the dots. Its dependence on the energy difference between the dots is derived.

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Introduction. Today quantum dot systems allow a detailed study of many physical phenomena, like Coulomb blockade [1,2], Kondo effects [3–5], interference effects [4]. As in these structures quantum states can be manipulated, they also may have an application in future quantum gates [6]. Quantum dot systems can typically be characterized by only a few parameters, which are experimentally controllable. Theoretically, these systems may be described by basic models, which capture the essential physics and can be investigated using standard methods of many-particle theory. However, if one deals with out-of-equilibrium situations and strong coupling, a theoretical analysis becomes difficult.

Such an out-of-equilibrium problem was recently studied, when the stationary tunnel current through a double quantum dot in a phonon bath was measured [7]. There the influence of the phonon environment was examined at low temperature. The thermal energy of the environment is always a source of unwanted transitions in quantum dot devices. Even at zero temperature spontaneous emission of phonons gives rise to inelastic transitions, i.e. they occur between dot states of nonequal energy. In the experiment in Ref. [7] the inelastic contribution to the tunnel current through the double dot was studied. A first theoretical interpretation of the experimental results focused on the interaction of the dots with the phonons, which is analogous to the spin-boson model [8]. They accounted for the coupling to the leads perturbatively and studied the electron-phonon problem using an approximation, which corresponds to the noninteracting blip approximation (NIBA) for the spin-boson model [9]. Thereby a better understanding of the shape of the inelastic current spectrum was achieved. It was shown that the interference of phonons interacting with the electron densities at the two dots leads to an oscillating structure in the current spectrum. However, a quantitative comparison with the experiment has not yet been possible. Especially, the unexpectedly large inelastic current of the experiment could not be explained.

In this paper we present a reliable solution for the stationary state of the double dot system. Since we deal with a nonequilibrium situation, where both the coupling to the phonon reservoir and the coupling to the leads have to be considered as strong, we use the recently developed real-time renormalization-group (RTRG) method. It has successfully been applied to both equilibrium [11,12] and nonequilibrium problems [13,14], including the spin-boson model [14,15]. Applying this approach to the double dot system both the electron and the phonon reservoirs are integrated out in a RG procedure, i.e. beyond perturbation theory. Thereby, the external voltage is accounted for properly, and the level broadening induced by the coupling to the leads is included in this method. Thus, we can also consider the case, where the energy difference between the dots is of the same order as the external voltage. Furthermore, we do not have to include an additional cutoff parameter, which simulates the level broadening generated by the leads. Moreover, this formalism may treat any form of dot-phonon interaction. Therefore, we are able to account for the full electron-phonon interaction, i.e. including interaction terms, which involve a tunneling between the dots ("offdiagonal interaction terms"). In the quantitative analysis the offdiagonal interaction terms lead to a strong dependence of the current on the extension of the wavefunctions within one dot. We find that the variation of this extension with the energy difference ϵ between the dot levels has to be accounted for. By fitting the result for the current with the experimental data, we obtain the width of the electron density within one dot as a function of ϵ. Furthermore, for the physically relevant situation, where the coupling of the dots to the leads and to the phonons is only moderately large, the RTRG approach does not deal with any parameter restriction. This is in contrast to the NIBA, which is valid only for sufficiently high temperature.

Model. Our model consists of two coupled quantum dots (l and r, respectively). Each dot is coupled to an electron reservoir with the chemical potentials μ_l and μ_r, see Fig. 1. We consider the case realized in the experi-

ment [7], where the external voltage $V = \mu_l - \mu_r$ is much smaller than the Coulomb charging energy U. Thus, due to Coulomb blockade the double dot cannot be charged with more than one additional electron. In the experiment [7] a strong magnetic field was applied perpendicular to the dots. Thus, we assume spin polarization here and omit the spin index. We denote the many-particle ground states, where an additional electron is in the left (right) dot, by $|l\rangle$ ($|r\rangle$) and neglect any excited states. Therefore, together with the uncharged ground state $|0\rangle$, there are only three possible states of the double dot. The total Hamiltonian $\bar{H}$ for the system can be written as a sum of the dot Hamiltonian, the contributions of the electron reservoirs and the phonon bath, and the interaction parts stemming from the coupling to the leads and the electron-phonon interaction:

$$\bar{H} = H_d + H_{res} + H_{ph} + H_{e-res} + H_{e-ph} \,. \qquad (1)$$

The dot Hamiltonian H_d reads

$$H_d = \epsilon_l |l\rangle\langle l| + \epsilon_r |r\rangle\langle r| + T_c \left(|l\rangle\langle r| + |r\rangle\langle l|\right) \,, \qquad (2)$$

where ϵ_l (ϵ_r) are the ground state energies of $|l\rangle$ ($|r\rangle$) and the coupling between the dots is described by the tunnel matrix element T_c. The reservoir contributions are given by

$$H_{res} = \sum_k \epsilon_k c_k^\dagger c_k + \sum_k \epsilon_k d_k^\dagger d_k \,, \qquad (3)$$

$$H_{ph} = \sum_q \omega_q a_q^\dagger a_q \,. \qquad (4)$$

Here, the operators $c_k^\dagger$ (c_k) create (annihilate) an electron with the energy ϵ_k in the left lead, whereas the creation (annihilation) operators $d_k^\dagger$ (d_k) refer to the right electron reservoir. Analogously, $a_q^\dagger$ (a_q) create (annihilate) a phonon with the wavevector $\vec{q}$ and the frequency ω_q. Here and throughout the paper we set $\hbar = 1$. The double dot is coupled to the external leads by the parameters V_k and W_k:

$$\begin{aligned} H_{e-res} &= \sum_k \left(V_k c_k |l\rangle\langle 0| + V_k^* |0\rangle\langle l| c_k^\dagger \right) \\ &+ \sum_k \left(W_k d_k |r\rangle\langle 0| + W_k^* |0\rangle\langle r| d_k^\dagger \right) \,. \qquad (5) \end{aligned}$$

The electron-phonon interaction consists of a diagonal part, which is characterized by the coupling constants α_q and β_q, and an offdiagonal contribution with the parameter γ_q:

$$\begin{aligned} H_{e-ph} &= \sum_q (\alpha_q |l\rangle\langle l| + \beta_q |r\rangle\langle r|)(a_q^\dagger + a_{-q}) \\ &+ \sum_q \gamma_q (|l\rangle\langle r| + |r\rangle\langle l|)(a_q^\dagger + a_{-q}) \,. \qquad (6) \end{aligned}$$

The above interaction coefficients are given by [8]

$$\alpha_q = \lambda_q \langle l| e^{i\vec{q}\vec{x}} |l\rangle \,, \qquad (7)$$

$$\beta_q = \lambda_q \langle r| e^{i\vec{q}\vec{x}} |r\rangle \,, \qquad (8)$$

$$\gamma_q = \lambda_q \langle l| e^{i\vec{q}\vec{x}} |r\rangle \,, \qquad (9)$$

where λ_q is the matrix element for the interaction of 2DEG electrons and phonons. The phonons are assumed to be three-dimensional acoustical phonons [7]. It then follows for the interaction [8]

$$|\lambda_q|^2 = g \frac{\pi^2 c_s^2}{V|\vec{q}|} \,, \qquad (10)$$

and the dispersion reads

$$\omega_q = c_s |\vec{q}| \,. \qquad (11)$$

Here, we introduced c_s as the speed of sound in the medium, V as the volume of the crystal, and the dimensionless coupling constant g [8]. For the evaluation of Eqs. (7) - (9) we model the electron densities $\rho_l(\vec{x})$ ($\rho_r(\vec{x})$) within one dot by Gaussians, which are peaked around the dot positions x_l ($\vec{x}_r = \vec{x}_l + \vec{d}$) with a width $|\Delta\vec{x}| = \sqrt{3/2}\,\sigma$:

$$\rho_{l(r)}(\vec{x}) = \left(\frac{1}{\pi\sigma^2}\right)^{3/2} e^{-\frac{(\vec{x}-\vec{x}_{l(r)})^2}{\sigma^2}} \,. \qquad (12)$$

The finite width σ leads to a high-energy cutoff $D = c_s/\sigma$ for the coefficients α_q, β_q and γ_q. We include this cutoff in an exponential form, so that we end up with the following interaction coefficients:

$$\alpha_q = \lambda_q e^{i\vec{q}\vec{x}_l} e^{-\frac{c_s|\vec{q}|}{2D}} \,, \qquad (13)$$

$$\beta_q = \lambda_q e^{i\vec{q}\vec{x}_r} e^{-\frac{c_s|\vec{q}|}{2D}} \,, \qquad (14)$$

$$\gamma_q = \lambda_q e^{i\vec{q}(\frac{\vec{x}_l+\vec{x}_r}{2})} e^{-\frac{|\vec{d}|D}{2c_s}} e^{-\frac{c_s|\vec{q}|}{2D}} \,. \qquad (15)$$

A simple form of the Hamiltonian, which shows the analogy with the spin-boson model, is obtained by shifting the bosonic field operators. One introduces the unitary transformation

$$U = \exp\left[\sum_q \left(\frac{\alpha_q + \beta_q}{2\omega_q} a_q^\dagger - \frac{\alpha_q^* + \beta_q^*}{2\omega_q} a_q\right)\right] \,, \qquad (16)$$

so that

$$U a_q U^\dagger = a_q - \frac{\alpha_q + \beta_q}{2\omega_q} \,. \qquad (17)$$

Thus, our final Hamiltonian $H = U\bar{H}U^\dagger$ reads

$$\begin{aligned} H &= H_0 + H_B + H_V \,, \qquad (18) \\ H_0 &= \frac{\epsilon}{2}\left(|l\rangle\langle l| - |r\rangle\langle r|\right) \end{aligned}$$

$$+T_c^{\text{eff}}\left(|l\rangle\langle r|+|r\rangle\langle l|\right)$$
$$+E\left(|l\rangle\langle l|+|r\rangle\langle r|-|0\rangle\langle 0|\right)\,, \qquad (19)$$
$$H_B=\sum_k \epsilon_k c_k^\dagger c_k+\sum_k \epsilon_k d_k^\dagger d_k+\sum_q \omega_q a_q^\dagger a_q\,, \qquad (20)$$
$$H_V=\sum_\mu :g_\mu j_\mu:\,, \qquad (21)$$

where we have used Eqs. (11) and (13) - (15). Furthermore, for simplicity we have set $(\epsilon_l+\epsilon_r)/2=0$ and have introduced the parameters

$$\epsilon=\epsilon_l-\epsilon_r\,, \qquad (22)$$
$$T_c^{\text{eff}}=T_c-2g\omega_d e^{-\frac{D}{2\omega_d}}\arctan\frac{D}{2\omega_d}\,, \qquad (23)$$
$$E=-\frac{g}{4}\left(D+\omega_d\arctan\frac{D}{\omega_d}\right) \qquad (24)$$

and

$$\omega_d=\frac{c_s}{|\vec{d}|}\,. \qquad (25)$$

Thus, the tunnel amplitude T_c has to be replaced by a smaller effective T_c^{eff}, which is due to the offdiagonal electron-phonon interaction. One already recognizes that the reduction of T_c strongly depends on the width of the electron densities $\sigma=|\vec{d}|\omega_d/D$. Finally, in view of the RTRG method, we have written the interaction part H_V as normal ordered products of local (dot) operators g_μ and environmental operators j_μ. They are defined by

$$g_{b_1}=\frac{1}{2}\left(|l\rangle\langle l|-|r\rangle\langle r|\right)\,, \qquad (26)$$
$$j_{b_1}=\sum_q\left(\alpha_q-\beta_q\right)\left(a_q^\dagger+a_{-q}\right)\,, \qquad (27)$$
$$g_{b_2}=\left(|l\rangle\langle r|+|r\rangle\langle l|\right)\,, \qquad (28)$$
$$j_{b_2}=\sum_q\gamma_q\left(a_q^\dagger+a_{-q}\right)\,, \qquad (29)$$
$$g_{b_3}=-\frac{1}{2}|0\rangle\langle 0|\,, \qquad (30)$$
$$j_{b_3}=\sum_q\left(\alpha_q+\beta_q\right)\left(a_q^\dagger+a_{-q}\right)\,, \qquad (31)$$
$$g_{+l}=g_{-l}^\dagger=|l\rangle\langle 0|\quad,\qquad j_{+l}=j_{-l}^\dagger=\sum_k V_k c_k\,, \qquad (32)$$
$$g_{+r}=g_{-r}^\dagger=|r\rangle\langle 0|\quad,\qquad j_{+r}=j_{-r}^\dagger=\sum_k W_k d_k\,. \qquad (33)$$
$$(34)$$

Therefore, the interaction index μ runs over the bosonic indices b_1,b_2,b_3 and the fermionic ones $+l,-l,+r,-r$. From Eqs. (18) - (21) the spin-boson model is recovered by omitting the electron reservoirs and the interaction with them, excluding the state $|0\rangle$ and accounting only for the bosonic interaction b_1. The latter corresponds to neglecting the offdiagonal electron-phonon (b_2) interaction.

RTRG approach. The RTRG approach is based on the formally exact kinetic equations

$$\langle I\rangle(t)=\mathrm{Tr}_0\left[\int_0^t dt'\,\Sigma_I(t-t')p(t')\right]\,, \qquad (35)$$
$$\dot p(t)+iL_0p(t)=\int_0^t dt'\,\Sigma(t-t')p(t')\,, \qquad (36)$$

where $\langle I\rangle(t)$ denotes the time-dependent expectation value of the current through the double dot, and $p(t)$ is the time-dependent reduced density matrix, which is the trace over the bath degrees of freedom Tr_B of the full density matrix. In contrast, Tr_0 denotes the trace over the local (dot) degrees of freedom. The bath degrees of freedom enter the equations via the integral kernels Σ and Σ_I, which are defined by sums over irreducible diagrams, see Ref. [16] for details. Introducing the Laplace transforms $f(z)=\int_0^\infty dt\,e^{izt}f(t)$ of the time-dependent functions $f(t)$ and using the identity $\lim_{t\to\infty}f(t)=-i\lim_{z\to0}zf(z)$ leads to the following expression for the stationary tunnel current I_{st}:

$$I_{st}=\mathrm{Tr}_0\left[\Sigma_I(z=0)p_{st}\right]\,, \qquad (37)$$

where the stationary reduced density matrix p_{st} is determined by

$$\left(L_0+i\Sigma(z=0)\right)p_{st}=0\,. \qquad (38)$$

In the following we outline only the main steps of the (i) derivation of Eqs. (35) and (36), and of the (ii) RTRG technique, by which Σ and Σ_I are calculated.

(i) The quantity of interest is the current through the double dot. It is given by the expectation value of the operator

$$I=ie\sum_k\left(V_k^*|0\rangle\langle l|c_k^\dagger-V_kc_k|l\rangle\langle 0|\right) \qquad (39)$$

with e being the elementary charge. For the calculation of $\langle I\rangle(t)$ we introduce the Liouvillian $L=[H,\cdot]_-$ and the superoperator $A_I=[\frac{i}{2}I,\cdot]_+$, where $[\cdot,\cdot]_{-(+)}$ denotes the (anti)commutator. The expectation value of the current can then be written as

$$\langle I\rangle(t)=-i\mathrm{Tr}\left[A_Ie^{-iLt}p(0)\rho_B^{eq}\right]\,. \qquad (40)$$

Here, we assumed a factorized density matrix at $t=0$, where $p(0)$ is the initial dot density matrix and ρ_B^{eq} is the equilibrium distribution of the electron reservoirs and the phonon bath. To evaluate the above expression the propagator $\exp(-iLt)$ is expanded in the interaction part $L_V=[H_V,\cdot]$. According to Eq. (21) this can be written as

$$L_V=\sum_{p\mu}:G_\mu^pJ_\mu^p:\,, \qquad (41)$$

where the index $p = \pm$ denotes, whether the interaction takes place on the forward or backward propagator, i.e.

$$G_\mu^+ = g_\mu \cdot \quad , \qquad G_\mu^- = \cdot\,(-g_\mu)\ , \tag{42}$$
$$J_\mu^+ = j_\mu \cdot \quad , \qquad J_\mu^- = \cdot\, j_\mu\,. \tag{43}$$

Correspondingly, we write $A_I = \sum_p :(A^p_{I+l} J^p_{+l} + A^p_{I-l} J^p_{-l}):$ with

$$A^+_{I+l} = \frac{e}{2}|l\rangle\langle 0|\cdot \quad , \qquad A^-_{I+l} = \cdot\,\frac{e}{2}|l\rangle\langle 0|\,, \tag{44}$$
$$A^+_{I-l} = -\frac{e}{2}|0\rangle\langle l|\cdot \quad , \qquad A^-_{I-l} = \cdot\,\left(-\frac{e}{2}|0\rangle\langle l|\right)\,. \tag{45}$$

The trace over the bath degrees of freedom can then be performed by application of Wick's theorem. In this way one obtains a series of terms, where vertices G^p_μ and $A^p_{I\pm l}$ of the local system are connected by pair contractions $\gamma^{pp'}_{\mu\mu'}(t) = \mathrm{Tr}_B\left[J^p_\mu J^{p'}_{\mu'}\rho^{eq}_B\right]$ of the bath. Denoting the sum over all irreducible diagrams, which contain the current superoperators $A^p_{I\pm l}$, by Σ_I then leads to Eq. (35), see Fig. 2. Similarly, Eq. (36) is obtained by introducing the object Σ, which is defined as the sum over all irreducible diagrams involving only the vertices G^p_μ.

(ii) The objects $\Sigma(z)$ and $\Sigma_I(z)$ are calculated by a renormalization group procedure. Short time scales of $\gamma^{pp'}_{\mu\mu'}(t)$ are integrated out first by introducing a short-time cutoff t_c into the correlation function $\gamma^{pp'}_{\mu\mu'}(t) \to \gamma^{pp'}_{\mu\mu'}(t, t_c)$. In each renormalization-group step, the time scales between t_c and $t_c + dt_c$ are integrated out, starting from $t_c = 0$ and ending at $t_c = \infty$. As a consequence, one generates RG equations for $\Sigma(z)$ ($\Sigma_I(z)$), L_0, G^p_μ, and the boundary vertex operators A^p_μ ($A^p_{I\mu}$)and B^p_μ (defined as the rightmost and leftmost vertex of the kernel $\Sigma(z)$ ($\Sigma_I(z)$)), for the explicit form of the RG equations see the Appendix.

Within the scheme of a perturbative RG analysis, the generation of multiple vertex superoperators is neglected here, as in Ref. [13,14]. Due to the small realistic value of g ($g = 0.05$ for GaAs) the neglecting of double- and higher-order vertex objects is justified and our approach leads to very reliable results, see Ref. [14], where we solved the spin-boson model for couplings up to $\alpha = g/2 \lesssim 0.1..0.2$. Furthermore, since also the coupling to the leads is treated nonperturbatively, the induced level broadening is accounted for properly. In contrast, in a perturbative analysis this effect has to be accounted for by an additional cutoff parameter. Also note that, in our method there is no restriction regarding the temperature, whereas the NIBA is only valid for the parameter regime $(\Delta_r^2 + \epsilon^2)^{1/2} \lesssim T$ [10]. Here, $\Delta_r = 2T_c(2T_c/D)^{\alpha/(1-\alpha)}$ is the renormalized tunnel amplitude of the spin-boson model. Finally, using the RTRG we are also able to account for the offdiagonal electron-phonon interaction, which is important for small D/ω_d, see Eqs. (23), (53) - (56).

Results. We solve the set of ordinary differential equations, Eqs. (64) - (68), numerically. The stationary tunnel current then follows from Eqs. (37) and (38). Our choice of the parameters corresponds to the experiment, where a GaAs structure was used at the temperature $T = 23mK = 1.98\mu eV$ with an external voltage $V = 140\mu eV$ [7]. For GaAs we have $c_s = 5000m/s$ and $g = 0.05$ [17]. The distance between the dots is estimated as $d = 200 \cdot 10^{-9}m$ [8], which leads to $\omega_d = 16.5\mu eV$.

The result for $T_c = \Gamma_l = \Gamma_r = 1\mu eV$, $D_l = D_r = 1meV$ and $D = 100\mu eV$ respectively $D = 150\mu eV$, which corresponds to the parameters studied in Ref. [8], is shown in Fig. 3. The external voltage V gives rise to a finite stationary tunnel current through the double dot. The elastic current can be seen around ϵ with a width depending on the coupling to the leads $\Gamma_l = \Gamma_r$ and the internal tunnel amplitude T_c. There the phonons do not participate in the tunnel process. Due to the coupling to the phonons there is also an inelastic current, where phonons are emitted ($\epsilon > 0$) respectively absorbed ($\epsilon < 0$) during the tunnel process. For increasing $\Gamma_{l(r)}$ the width of the elastic current grows, while an increased coupling constant g leads to a larger inelastic current. The effect of the finite voltage V can be seen in Fig. 3, where for $\epsilon > V$ the tunnel current drops to zero. Furthermore, Fig. 3 shows that the offdiagonal interaction leads to a larger inelastic current. This effect is increased with decreasing D, see also Eqs. (53) - (56). We will see below (Fig. 5) that, due to Eq. (23), the offdiagonal interaction also suppresses the elastic current. Eventually, in Fig. 3 one also recognizes the oscillations stemming from the interference of the phonons interacting with the two dots [8].

Let us now study the current quantitatively in comparison with the experiment. For this it is necessary to choose realistic parameter values for T_c, $\Gamma_{l(r)}$, D and $D_{l(r)}$. From changing the bias polarity in the experiments the ratio $\Gamma_r/\Gamma_l \approx 0.5..1$ was found [7]. To determine the couplings T_c and Γ_r, the experimental data are compared with the result of Stoof and Nazarov [18]

$$I_{st} = \frac{T_c^2 \Gamma_r}{T_c^2(2 + \Gamma_r/\Gamma_l) + \Gamma_r^2/4 + \epsilon^2}\,, \tag{46}$$

which is valid for no electron-phonon interaction ($g = 0$). A good agreement of the elastic current is found for $T_c = 0.124\mu eV$ and $\Gamma_l = \Gamma_r = 3.5\mu eV$, see Fig. 4.

However, due to the absent electron-phonon interaction the influence of the finite extension of the electron densities within the dots is also disregarded in the Stoof-Nazarov result. In contrast, our method accounts for this extension by the high-energy cutoff D. From Eq. (23) we see that for a finite D the tunnel amplitude is effectively reduced. Therefore, the Stoof-Nazarov result underestimates the value of T_c.

From the large inelastic current in the experiment one can conclude, that in fact a finite value of D was re-

alized. It turns out that $T_c \approx 0.375$ allows sensible fits. First, in Fig. 5 our results for $T_c = 0.375\mu eV$, $\Gamma_l = \Gamma_r = 3.5\mu eV$, $D_l = D_r = 1meV$ and $D = 70\mu eV$ respectively $D = 100\mu eV$ are compared with the experiment. One recognizes that with decreasing D the larger overlap of the dots' wavefunctions leads to a stronger impact of the offdiagonal electron-phonon interaction. The elastic current is suppressed, whereas the inelastic current is increased. The deviations from the experiment show, that there is an ϵ dependence of the width of the electron densities, which we have to account for in order to achieve agreement. In Fig. 6 we show a fit of the width $\sigma = d\omega_d/D$, which is based on the experimental results for I_{st}. One recognizes that for larger absolute values of ϵ the electron densities are more sharply peaked. The asymmetry is due to the finite external voltage V. For $\epsilon < 0$ the state $|l\rangle$ lies in a deep potential well, thus this energetic separation of the two quantum dot levels and the leads causes a very small overlap of the wavefunctions within the dots. On the other hand for $\epsilon > 0$ neither dot level lies in a deep potential well, however, an increasing energetic separation ϵ again leads to more sharply defined electron densities.

In Fig. 5 one also recognizes that the structure on the emission side ($\epsilon > 0$) of the current spectrum observed in the experiment does not stem from interference effects of the phonons. In fact, the oscillations generated by this interference occur on a much larger energy scale than the structure found in the experimental curve. In contrast our results show that the bump on the emission side of the current spectrum is due to another mechanism: on the one hand we see in Fig. 5 that for small $\epsilon > 0$ the inelastic current grows with increasing ϵ. On the other hand however, the electron densities are simultaneously sharpened, so that the cutoff D is increased. For larger ϵ this again reduces the inelastic current.

In summary, we have applied the RTRG method to the coupled quantum dot system in a phonon bath in nonequilibrium. By accounting for both the coupling to the leads and the coupling to the environmental phonons nonperturbatively we achieved a reliable solution for the stationary tunnel current. For the first time both the elastic and the inelastic current of the experiment could quantitatively be reproduced. Our analysis shows the importance of the finite width σ of the electron densities within one dot, and for the experiment, the dependence of σ on the energy difference ϵ between the dots was calculated.

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Appendix

To obtain the explicit form of the RG equations one has to make a choice of the t_c dependence of the bath contractions $\gamma^{pp'}_{\mu\mu'}(t,t_c)$. Since the bosonic bath contractions $\gamma^{pp'}_{b_j b_k}(t)$ $(j,k = 1,2,3)$ correspond to those of the spin-boson model, we choose the t_c dependence as in Refs. [14,15]:

$$\gamma^{pp'}_{b_j b_k}(t,t_c) = \frac{d}{dt}\left(\tilde{R}_{jk}(t)\Theta(t-t_c)\right) + ip' S_{jk}(t)\Theta(t-t_c)\,. \tag{47}$$

The functions $\tilde{R}_{jk}(t)$ and $S_{jk}(t)$ are defined by

$$\gamma^{pp'}_{b_j b_k}(t) = \frac{d}{dt}\tilde{R}_{jk}(t) + ip' S_{jk}(t)\,. \tag{48}$$

From Eqs. (13) - (15) we obtain

$$\tilde{R}_{11}(t) = g\,\mathrm{Re}\Big[\pi T\coth\left(\pi T(t-i/D)\right) + \frac{\omega_d}{2}\ln\left(\frac{\sinh\left(\pi T(t-1/\omega_d - i/D)\right)}{\sinh\left(\pi T(t+1/\omega_d - i/D)\right)}\right)\Big]\,, \tag{49}$$

$$S_{11}(t) = g\,\mathrm{Im}\Big[\frac{1/\omega_d^2}{\left((t-i/D)^2 - 1/\omega_d^2\right)(t-i/D)^2}\Big]\,, \tag{50}$$

$$\tilde{R}_{12} = \tilde{R}_{21} = S_{12} = S_{21} = 0\,, \tag{51}$$

$$\tilde{R}_{13} = \tilde{R}_{31} = S_{13} = S_{31} = 0\,, \tag{52}$$

$$\tilde{R}_{22}(t) = \frac{g}{2}e^{-D/\omega_d}\,\mathrm{Re}\Big[\pi T\coth\left(\pi T(t-i/D)\right)\Big]\,, \tag{53}$$

$$S_{22}(t) = -\frac{g}{2}e^{-D/\omega_d}\,\mathrm{Im}\Big[\frac{1}{(t-i/D)^2}\Big]\,, \tag{54}$$

$$\tilde{R}_{23}(t) = \tilde{R}_{32}(t) = -g\,\mathrm{Re}\Big[\omega_d e^{-D/2\omega_d} \times\ln\left(\frac{\sinh\left(\pi T(t-1/2\omega_d - i/D)\right)}{\sinh\left(\pi T(t+1/2\omega_d - i/D)\right)}\right)\Big]\,, \tag{55}$$

$$S_{23}(t) = S_{32}(t) = -g\,\mathrm{Im}\Big[\frac{e^{-D/2\omega_d}}{\left((t-i/D)^2 - 1/4\omega_d^2\right)}\Big]\,, \tag{56}$$

$$\tilde{R}_{33}(t) = g\,\mathrm{Re}\Big[\pi T\coth\left(\pi T(t-i/D)\right) - \frac{\omega_d}{2}\ln\left(\frac{\sinh\left(\pi T(t-1/\omega_d - i/D)\right)}{\sinh\left(\pi T(t+1/\omega_d - i/D)\right)}\right)\Big]\,, \tag{57}$$

$$S_{33}(t) = g\,\mathrm{Im}\Big[\frac{1/\omega_d^2 - 2(t-i/D)^2}{\left((t-i/D)^2 - 1/\omega_d^2\right)(t-i/D)^2}\Big]\,. \tag{58}$$

Here, we assumed that the high-energy cutoff D is much larger than the temperature T. In Eqs. (53) - (56) one again recognizes that the influence of the offdiagonal interaction strongly depends on the width of the electron densities $\sigma = |\vec{d}|\omega_d/D$. The fermionic contractions can be written as

$$\gamma^{pp'}_{\eta f\eta' f'}(t) = \delta_{\eta,-\eta'}\delta_{f,f'}\left(\frac{1}{2}\left(\gamma_{\eta f}(t) + \gamma_{-\eta f}(-t)\right) + \frac{p'}{2}\left(\gamma_{\eta f}(t) - \gamma_{-\eta f}(-t)\right)\right)\,, \tag{59}$$

where the indices η and f run over $\pm$ and l, r. We introduce

$$\Gamma_l(\epsilon) = 2\pi \sum_k |V_k|^2 \delta(\epsilon - \epsilon_k)\,, \tag{60}$$

$$\Gamma_r(\epsilon) = 2\pi \sum_k |W_k|^2 \delta(\epsilon - \epsilon_k)\,, \tag{61}$$

so that we obtain

$$\gamma_{\eta f}(t) = \frac{-iT\Gamma_f e^{-i\eta\mu_f t}}{2\sinh\left(\pi T(t - i/D_f)\right)}\,. \tag{62}$$

Here, we introduced a bandwidth D_f of the reservoir f and assumed, that $\Gamma_f(\epsilon) \approx$ const. holds. The cutoff-dependence of the fermionic contractions is chosen as

$$\gamma^{pp'}_{\eta f \eta' f'}(t, t_c) = \gamma^{pp'}_{\eta f \eta' f'}(t)\Theta(t - t_c)\,. \tag{63}$$

The final RG equations then read (for a more detailed derivation we refer to Ref. [16])

$$\begin{aligned}
\frac{d\Sigma_{(I)}}{dt_c} =& \sum_{p_1,p_2,j,k} -i\left(\tilde{R}_{jk}(t_c) A^{p_1}_{(I)b_j}(t_c)(L_0 - z)\right.\\
&\left. + p_2 S_{jk}(t_c) A^{p_1}_{(I)b_j}(t_c)\right) B^{p_2}_{b_k}\\
&- \sum_{p_1,p_2,\eta,f} \gamma^{p_1p_2}_{-\eta f \eta f}(t_c)\hat{\sigma}^{p_1p_2} A^{p_1}_{(I)-\eta f}(t_c) B^{p_2}_{\eta f}\,,
\end{aligned} \tag{64}$$

$$\begin{aligned}
\frac{dL_0}{dt_c} =& \sum_{p_1,p_2,j,k} \left(\tilde{R}_{jk}(t_c)[G^{p_1}_{b_j}(t_c), L_0]\right.\\
&\left. + p_2 S_{jk}(t_c) G^{p_1}_{b_j}(t_c)\right) G^{p_2}_{b_k}\\
&- i \sum_{p_1,p_2,\eta,f} \gamma^{p_1p_2}_{-\eta f \eta f}(t_c)\hat{\sigma}^{p_1p_2} G^{p_1}_{-\eta f}(t_c) G^{p_2}_{\eta f}\,,
\end{aligned} \tag{65}$$

$$\begin{aligned}
\frac{dG^p_\mu}{dt_c} =& \sum_{p_1,p_2,j,k} \tilde{R}_{jk}(t_c)\left(G^{p_1}_{b_j}(t_c) G^p_\mu - G^p_\mu G^{p_1}_{b_j}(t_c)\right) G^{p_2}_{b_k}\\
&- i\int_0^{t_c} dt\,\left(G^{p_1}_{b_j}(t) G^p_\mu - G^p_\mu G^{p_1}_{b_j}(t)\right)\\
&\times\left(\tilde{R}_{jk}(t_c)[L_0, G^{p_2}_{b_k}(t - t_c)]\right.\\
&\left. + p_2 S_{jk}(t_c) G^{p_2}_{b_k}(t - t_c)\right)\\
&+ \sum_{p_1,p_2,\eta,f} \gamma^{p_1p_2}_{-\eta f \eta f}(t_c)\int_0^{t_c} dt\,\left(G^p_\mu \hat{\sigma}^{p_1p_2} G^{p_1}_{-\eta f}(t)\right.\\
&\left. - \eta^{pp_2}_\mu \hat{\sigma}^{p_1p_2} G^{p_1}_{-\eta f}(t) G^p_\mu\right) G^{p_2}_{\eta f}(t - t_c)\,,
\end{aligned} \tag{66}$$

$$\begin{aligned}
\frac{dA^p_{(I)\mu}}{dt_c} =& \sum_{p_1,p_2,j,k} \tilde{R}_{jk}(t_c)\left(A^{p_1}_{(I)b_j}(t_c) G^p_\mu\right.\\
&\left. - A^p_{(I)\mu} G^{p_1}_{b_j}(t_c)\right) G^{p_2}_{b_k}\\
&- i\int_0^{t_c} dt\,\left(A^{p_1}_{(I)b_j}(t) G^p_\mu - A^p_{(I)\mu} G^{p_1}_{b_j}(t)\right)\\
&\times\left(\tilde{R}_{jk}(t_c)[L_0, G^{p_2}_{b_k}(t - t_c)]\right.\\
&\left. + p_2 S_{jk}(t_c) G^{p_2}_{b_k}(t - t_c)\right)\\
&+ \sum_{p_1,p_2,\eta,f} \gamma^{p_1p_2}_{-\eta f \eta f}(t_c)\\
&\times\int_0^{t_c} dt\,\left(A^p_{(I)\mu}\hat{\sigma}^{p_1p_2} G^{p_1}_{-\eta f}(t)\right.\\
&\left. - \eta^{pp_2}_\mu \hat{\sigma}^{p_1p_2} A^{p_1}_{(I)-\eta f}(t) G^p_\mu\right) G^{p_2}_{\eta f}(t - t_c)\,,
\end{aligned} \tag{67}$$

$$\begin{aligned}
\frac{dB^p_\mu}{dt_c} =& \sum_{p_1,p_2,j,k} \tilde{R}_{jk}(t_c) G^{p_1}_{b_j}(t_c) G^p_\mu B^{p_2}_{b_k} - i\int_0^{t_c} dt\, G^{p_1}_{b_j}(t)\\
&\times G^p_\mu\left(\tilde{R}_{jk}(t_c)(L_0 - z) B^{p_2}_{b_k}(t - t_c)\right.\\
&\left. + p_2 S_{jk}(t_c) B^{p_2}_{b_k}(t - t_c)\right)\\
&- \sum_{p_1,p_2,\eta,f} \gamma^{p_1p_2}_{-\eta f \eta f}(t_c)\\
&\times\int_0^{t_c} dt\,\eta^{pp_2}_\mu \hat{\sigma}^{p_1p_2} G^{p_1}_{-\eta f}(t) G^p_\mu B^{p_2}_{\eta f}(t - t_c)\,.
\end{aligned} \tag{68}$$

The interaction picture is defined by $G^p_\mu(t) = e^{iL_0 t} G^p_\mu e^{-iL_0 t}$, $A^p_{(I)\mu}(t) = e^{izt} A^p_{(I)\mu} e^{-iL_0 t}$, and $B^p_\mu(t) = e^{iL_0 t} B^p_\mu e^{-izt}$. The function $\eta^{pp'}_\mu$ and the superoperator $\hat{\sigma}^{pp'}$ account for additional signs arising from the commutation of fermionic field operators. They are given by

$$\eta^{pp'}_\mu = \begin{cases} -pp' & \text{for } \mu \text{ fermionic} \\ 1 & \text{else} \end{cases}\,, \tag{69}$$

$$\left(\hat{\sigma}^{pp'}\right)_{ss',ss'} = \begin{cases} pp' & \text{for } N_s - N_{s'} = \text{odd} \\ 1 & \text{for } N_s - N_{s'} = \text{even} \end{cases}\,. \tag{70}$$

We also note, that in the limit of large D_l and D_r the first part of the fermionic contractions in Eq. (59) can be written as

$$\frac{1}{2}\left(\gamma_{\eta f}(t) + \gamma_{-\eta f}(-t)\right) = \frac{\Gamma_f}{2}\delta(t)\,. \tag{71}$$

Thus, for $D_l, D_r \to \infty$, this contribution to the differential equations Eqs. (64) - (68) can be incorporated in the initial conditions.

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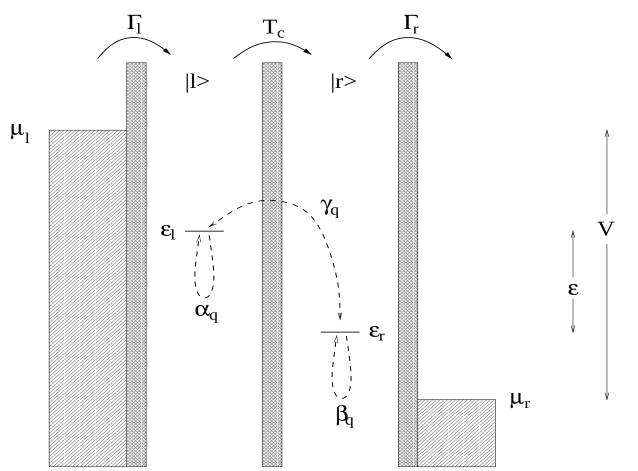

FIG. 1. The double quantum dot may be charged with one additional electron in the left or right dot. The corresponding states $|l\rangle$ and $|r\rangle$ are coupled by the tunnel amplitude T_c. The couplings to the leads are given by $\Gamma_{l(r)}$. The energy difference between the quantum dots is $\epsilon = \epsilon_l - \epsilon_r$, and there is an external voltage $V = \mu_l - \mu_r$. The interaction with the acoustical phonons (dashed lines), consists of a diagonal part with the coefficients α_q and β_q, and an offdiagonal part with the constants γ_q.

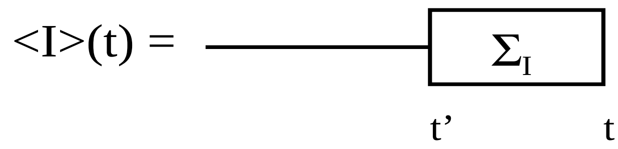

FIG. 2. Diagrammatic expression for $\langle I \rangle(t)$. The two lines of the Keldysh contour are put together to one line. The irreducible diagrams in Σ_I include the leftmost vertex superoperator at the time point t' and the current superoperator A_I at the time point t. Σ_I acts on $p(t')$, which is represented by the horizontal line.

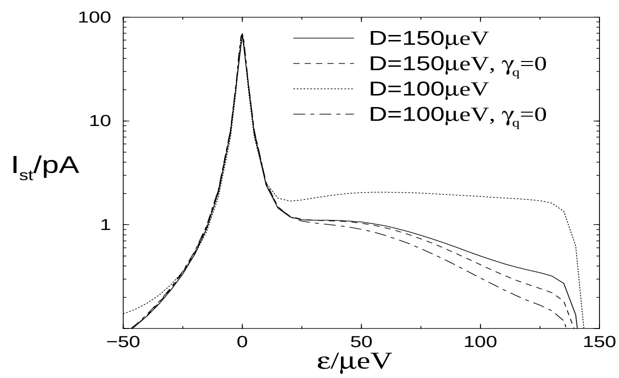

FIG. 3. Stationary tunnel current as a function of ϵ for $T_c = \Gamma_l = \Gamma_r = 1\mu eV$ and $D_l = D_r = 1meV$. and $D = 100\mu eV$ respectively $D = 150\mu eV$. Solid line: $D = 150\mu eV$. Dashed line: $D = 150\mu eV$, $\gamma_q = 0$. Dotted line: $D = 100\mu eV$. Dot-dashed line: $D = 100\mu eV$, $\gamma_q = 0$.

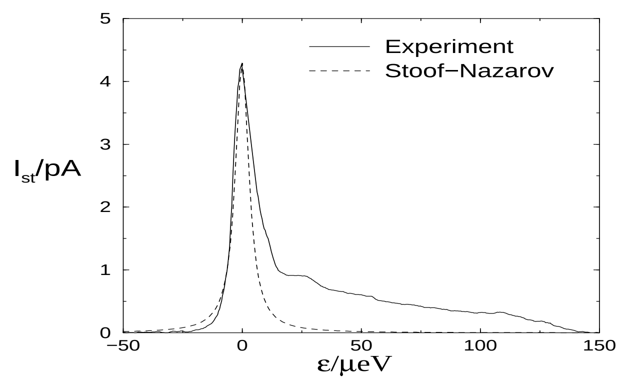

FIG. 4. Stationary tunnel current as a function of ϵ. Solid line: Experiment. Dashed line: Stoof-Nazarov result for the case of no electron-phonon interaction, with the parameters $T_c = 0.124\mu eV$ and $\Gamma_l = \Gamma_r = 3.5\mu eV$.

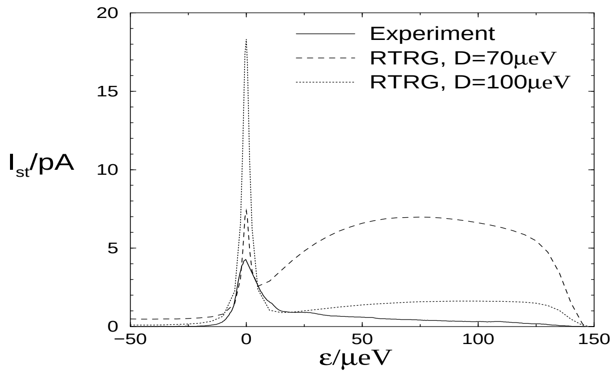

FIG. 5. Stationary tunnel current as a function of ϵ. Solid line: Experiment. Dashed line: RTRG with $T_c = 0.375\mu eV$, $\Gamma_l = \Gamma_r = 3.5\mu eV$, $D_l = D_r = 1meV$ and $D = 70\mu eV$. Dotted line: RTRG with $T_c = 0.375\mu eV$, $\Gamma_l = \Gamma_r = 3.5\mu eV$, $D_l = D_r = 1meV$ and $D = 100\mu eV$.

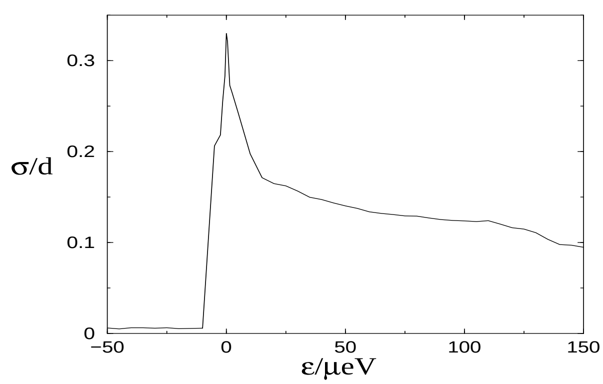

FIG. 6. The width of the electron density within one dot, σ, as a function of ϵ. $T_c = 0.375\mu eV$, $\Gamma_l = \Gamma_r = 3.5\mu eV$ and $D_l = D_r = 1meV$.